TN352_Mcr_implementation_adapt_010710

IMPLEMENTATION OF CRACKING MOMENT AND DETERMINATION OF Ie IN ADAPT

First draft

The topic of cracking moment of a section and its impact on a member's stiffness and consequently deflection have been treated in the following ADAPT Technical Notes.

- TN349 defines the cracking moment Mcr to initiate crack at a section
- TN310 outlines the calculation steps for the reduced second moment of area (Ie), once a section is cracked
- TN351 Calculation of Cracked Deflection

The thrust of the Technical Notes listed above is to adhere to the rules of mechanics of solids, as much as practical, while extending the bounds of the empirical formula commonly commonly used for conventionally reinforced concrete to cover prestressed members. TN351 concludes with a suggestion and a flow chart for the implementation for a general case of prestressed members.

With respect to the specifics of ADAPT programs, the suggested procedure in the referred Technical Notes needs to be modified as described in this Technical Note, in order to achieve an expeditious implementation, without compromising the inherent features of the empirical formula. The following are the assumptions made and details for implementation.

- Prestressing is considered as an integral part of a member, similar to a member's geometry and material properties.
- The impact of prestressing at any section on a member is (i) an average precompression (P/A) and (ii) a moment (Mp).
- The cracking moment of a prestressed member under all loading conditions, among other parameters, depend on both P/A and Mp. Hence, in all load combinations where the initiation of cracking and reduction in stiffness is sought, both P/A and Mp must be included with a load factor of 1.

BACKGROUND

For conventionally reinforced concrete, the reduction in flexural stiffness due to cracking is approximated through substitution of the original second moment of area of the section (Ig) by a reduced value (Ie) using the following empirical formula.

$$Ie = (Mcr/Ma)^3 Ig + [1 - (Mcr/Ma)^3] Icr \leq Ig$$

Where:

Icr = second moment of area of cracked section;
Ie = effective second moment of area;
Ma = applied moment at the section, where cracking occurs; and
Mcr = moment that initiates cracking in the section.

support@adaptsoft.com www.adaptsoft.com
ADAPT Corporation, Redwood City, California, USA, Tel: (650) 306-2400 Fax (650) 306 2401
ADAPT International Pvt. Ltd, Kolkata, India, Tel: 91 33 302 86580 Fax: 91 33 224 67281

This empirical relationship may be extended to apply to prestressed members, if the cracking stress of concrete (ft) is assume to have been enhanced by the magnitude of (P/A). In this case a pseudo-cracking moment can be defined as follows:

$$M'cr = S\,(ft + P/A)$$

The introduction of pseudo- cracking moment is accompanied by a pseudo- applied moment M'a, in which the prestressing is always included with a factor of 1.

M'a = moment to externally applied loads + Mp

Where Mp is the moment due to prestressing. It includes both the primary and secondary (hyperstatic actions of prestressing). The relationship to use is:

$$Ie = (M'cr/M'a)^3 Ig + [\,1 - (M'cr/M'a)^3\,]Icr \le Ig$$

The following describes the steps to arrive at the cracked deflection, once the pseudo-cracking moment (M'cr) and the procedure for the calculation of the equivalent second moment of area (Ie) are known.

Given:
Member (Fig. 1) under an applied load Q and post-tensioning force PT

Required:
What are the steps to calculate the deflection of this member ? (i) If it does not crack, and (ii) if it cracks.

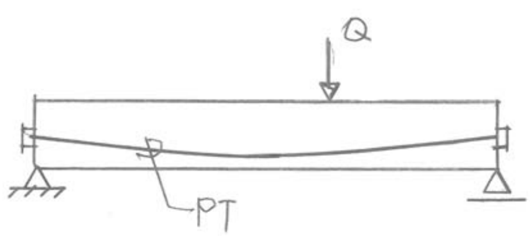

FIGURE 1 – PRESTRESSED MEMBER UNDER EXTERNAL LOAD Q

BACKGROUND

- Deflection is calculated using the following relationship:

$$\{u\} = [A]^{-1} * \{p\}$$

Where
{u} = displacement (deflection);
[A] = stiffness matrix of the structure' and
{p} = applied load.

The distribution of moment in a member is derived from {u}. In common engineering practice gross cross-sectional properties are used to determine the moment, even when a section is cracked. The calculated moment is referred to as {Ma}.

- The system matrix of an uncracked structure is based on its gross cross section(Ig). We refer to this system matrix as follows:

  [Ag] = system matrix of uncracked structure.

- Likewise the system matrix of a cracked structure is based on Ie. We refer to this system matrix as follows:

  [Ae] = system matrix of cracked structure.

- The process of deflection calculation with allowance for cracking is as follows:

  - Determine the pseudo- cracking moment of the member (M'cr). This includes the precompression (P/A) from prestressing.
  - Determine the externally applied moments This includes prestressing with factor 1.
  - If the pseudo- applied moments (M'a) at all locations are less than the cracking moment, the member does not crack. Determine the deflection using the common procedure (refer to flow chart below).
  - If the pseudo-externally applied moments (M'a) at any location exceed the pseudo cracking moment (M'cr) at that location consider the location cracked. Dtermine the reduction in stiffness and form a stiffness matrix based on the reduced stiffness [Ae].
  - Using the reduced stiffness, apply the load {Q+PT} to determine the deflection.

The following flow chart describes the steps.

FLOW CHART FOR CALCULATION OF DEFLECTIONS